Homework I

Due: Sep. 28. (Tue.) 23:59 PM

I. Remark

- Reading materials: Ch 1.1-1.7 in the textbook.
- Don't write just an answer. Please describe enough processes to justify your answer (Korean is also OK!!).
- "unique solution" = "only one solution"
- Don't just copy your colleagues' solution, If then, your score will be zero.
- Well begun is half done!!!

II. Problem Set

1) Find the general solution of the systems whose augmented matrix is given in

$$\begin{bmatrix} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{bmatrix}$$

2) Determine if the system is consistent.

$$\begin{aligned} x_1 - 2x_4 &= -3 \\ 2x_2 + 2x_3 &= 0 \\ x_3 + 3x_4 &= 1 \\ -2x_1 + 3x_2 + 2x_3 + x_4 &= 5 \end{aligned}$$

3) Suppose the coefficient matrix of a system of linear equations has a pivot position in every row. Is the system consistent? Justify your answer.

4) A system of linear equations with fewer equations than unknowns (variables) is called an underdetermined system. Suppose that the system is consistent. Do the system has an infinite number of solutions? Justify your answer.

5) Determine if $\mathbf{b}$ is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$.

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$$

6) For what value(s) of h is $\mathbf{b}$ in the plane spanned by $\mathbf{a}_1$ and $\mathbf{a}_2$?

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -4 \\ -1 \\ h \end{bmatrix}$$

7) Do the columns of B span $\mathbb{R}^4$? Does the equation $B\mathbf{x} = \mathbf{y}$ have a solution for every $\mathbf{y}$ in $\mathbb{R}^4$?

$$B = \begin{bmatrix} 1 & 3 & 2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix}$$

8) Suppose A is a 3×3 matrix and $\mathbf{b}$ is a vector in $\mathbb{R}^3$ with the property that $A\mathbf{x} = \mathbf{b}$ has a unique solution. Explain why the columns of A must span $\mathbb{R}^3$.

9) Determine if the system has a nontrivial solution.

$$\begin{aligned} 2x_1 - 5x_2 + 8x_3 &= 0 \\ -2x_1 - 7x_2 + x_3 &= 0 \\ 4x_1 + 2x_2 + 7x_3 &= 0 \end{aligned}$$

10) Mark each statement True of False. Justify each answer.
   a) The columns of a matrix A are linearly independent if the equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution.
   b) The columns of any 4×6 matrix are linearly dependent.
   c) If $\mathbf{z}$ is in Span$\{\mathbf{w}, \mathbf{x}, \mathbf{y}\}$, then $\{\mathbf{w}, \mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent.