

Homework II

Due: Oct. 16. (Sat) 23:59 PM

I. REMARK

- Reading materials: Ch 1-3 in the textbook.
- Don't write just an answer. Please describe enough processes to justify your answer (Korean is also OK!!).
- Be healthy!!!

II. PROBLEM SET

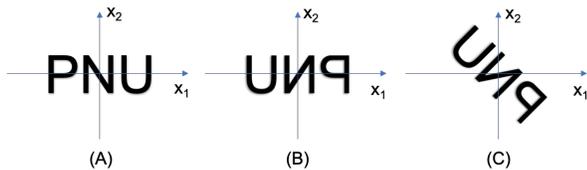
1) Vectors are given as

$$\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{y}_1 = \begin{bmatrix} -2 \\ 5 \end{bmatrix}, \mathbf{y}_2 = \begin{bmatrix} 1 \\ 6 \end{bmatrix}, \mathbf{z} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}.$$

Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation that maps \mathbf{e}_1 into \mathbf{y}_1 and maps \mathbf{e}_2 into \mathbf{y}_2 . Find the images of \mathbf{z} .

2) Show that the transformation T defined by $T(x_1, x_2) = (|4x_1 - 2x_2|, 3x_2)$ is whether linear or nonlinear.

3) $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ first reflects points through the vertical x_2 -axis ((A) to (B)) and then rotates points through $\pi/4$ radian, clockwise ((B) to (C)). Find the standard matrix of T . Also, find the matrix of T^{-1} .



4) Find A^{-1} , A^T , $(A^T)^{-1}$ and AB .

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 1 & 0 \end{bmatrix}.$$

5) The matrix S is partitioned as

$$S = \begin{bmatrix} I & 0 & 0 \\ C & I & 0 \\ A & B & I \end{bmatrix}.$$

Find S^{-1} .

6) Solve the equation $A\mathbf{x} = \mathbf{b}$ by using the LU factorization given for A .

$$A = \begin{bmatrix} 3 & -7 & -2 \\ -3 & 5 & 1 \\ 6 & -4 & 0 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -7 \\ 5 \\ 2 \end{bmatrix}$$

7) A is in $\mathbb{R}^{n \times n}$. Explain why the columns of A^2 span \mathbb{R}^n whenever the columns of A are linearly independent.

8) Find a basis for Col A and a basis for Nul A . What is Rank A ?

$$A = \begin{bmatrix} 4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -3 \end{bmatrix}$$

9) Justify whether H is a subspace of \mathbb{R}^3 or not. If it is, what is the dimension of the subspace?

$$H = \text{span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \right\}$$

10) Find the coordinate vector $[\mathbf{x}]_C$.

$$B = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}, C = \left\{ \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}, [\mathbf{x}]_B = \begin{bmatrix} 3 \\ 2 \end{bmatrix},$$